P35

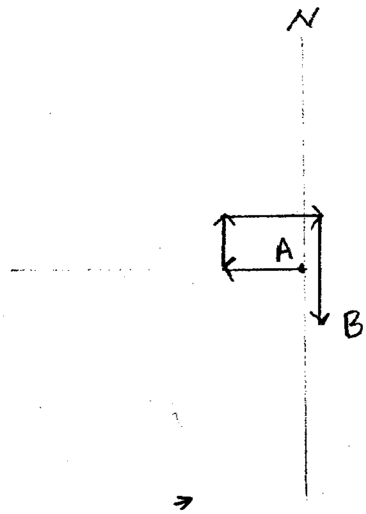
[1] $\overrightarrow{BC} = \overrightarrow{AD}$, $\overrightarrow{BA} = \overrightarrow{CD}$

P36

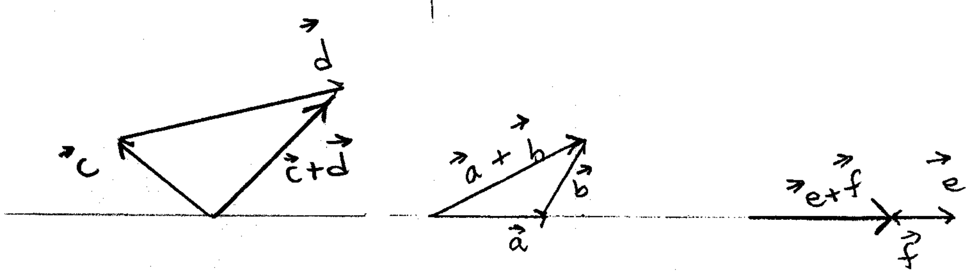
[2] EQUAL: $\vec{d} = \vec{f}$. INVERSE: $\vec{b}, \vec{e}$

P37

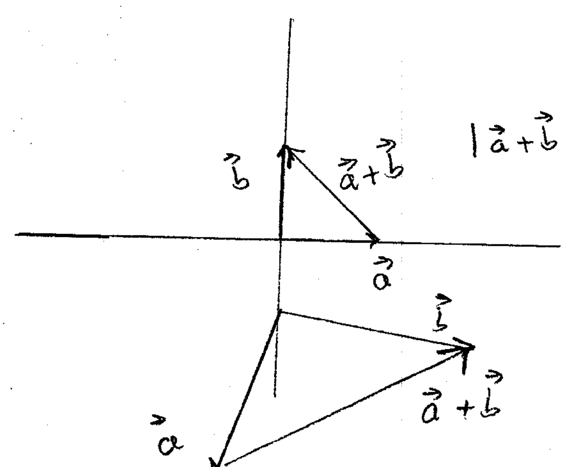
[1]



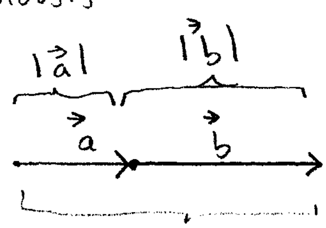
[2]



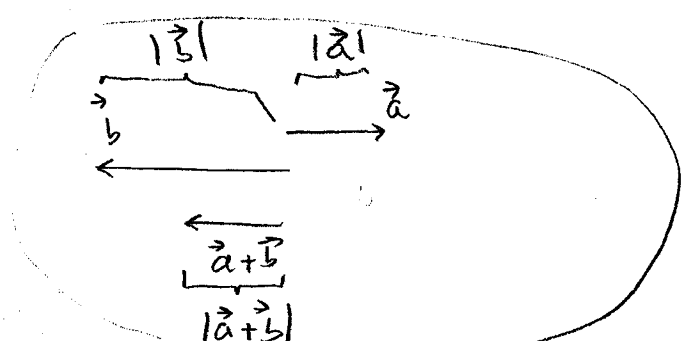
[3]



$|\vec{a}+\vec{b}|$ obviously less than $|\vec{a}| + |\vec{b}|$.



$|\vec{a}+\vec{b}|$ equal $|\vec{a}|+|\vec{b}|$



Hard to imagine how $|\vec{a}|+|\vec{b}|$ could be less than $|\vec{a}+\vec{b}|$. It can not.

P38

[4] $\vec{a}$ followed by $\vec{b}$ equivalent to $\vec{c}$

$\vec{b}$ " " $\vec{a}$ " " $\vec{c}$

$\vec{a}$ followed by ($\vec{b}$ followed by $\vec{c}$) is $\overrightarrow{AD}$

($\vec{a}$ followed by $\vec{b}$) followed by $\vec{c}$ is $\overrightarrow{AD}$

[5] Prove $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{O}$

Proof

$$\begin{aligned} LHS &= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} \\ &= \overrightarrow{AC} + \overrightarrow{CA} \\ &= \overrightarrow{AC} - \overrightarrow{AC} \\ &= \vec{O} \end{aligned}$$

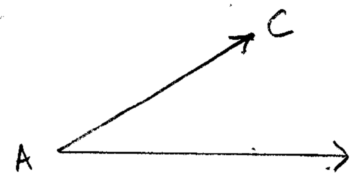
P39

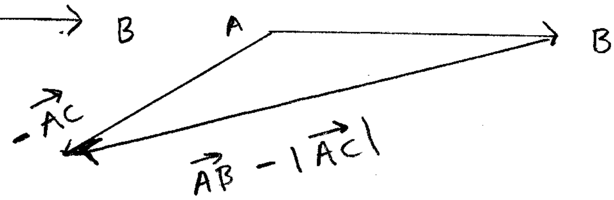
[6] Let $\vec{a} - \vec{b} = \vec{x}$. check $\vec{x}$ such that $\vec{b} + \vec{x} = \vec{a}$.

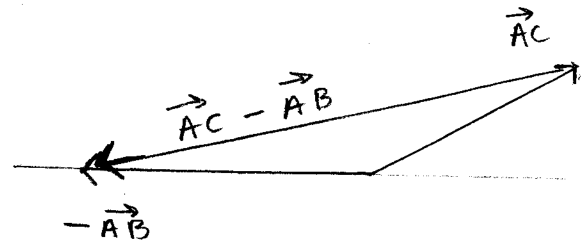
$$\begin{aligned} LHS &= \vec{a} - \vec{b} \\ &= \vec{a} + (-\vec{b}) \quad \text{since} \\ &= \vec{b} + \vec{x} - \vec{b} \quad \text{assoc and comm.} \\ &= \vec{x} \\ &= RHS \end{aligned}$$

P39, ctd

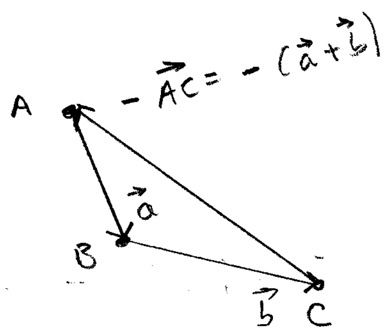
[7]



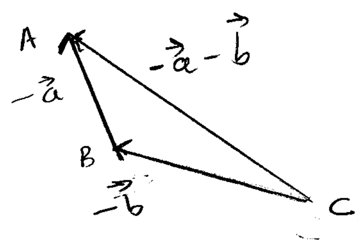




[8]



$\left.\begin{array}{l} \vec{a} = \vec{AB} \\ \vec{b} = \vec{BC} \end{array}\right\}$ then $-(\vec{a}+\vec{b}) = -\vec{AC}$

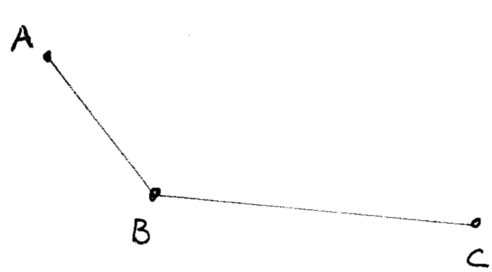


Trip from C to B to A is

$-\vec{BC} - \vec{AB} = -\vec{AC}$

$\vec{CB} + \vec{BA} = \vec{CA}$

Think,



$\vec{AB} + \vec{BC} = \vec{AC}$

$-(\vec{AB} + \vec{BC}) = -\vec{AC} = \vec{CA}$

and

$$\begin{aligned} -\vec{AB} - \vec{BC} &= \vec{BA} + \vec{CB} \\ &= \vec{CB} + \vec{BA} \\ &= \vec{CA} \\ &= -\vec{AC} \end{aligned}$$

P41

[1] $\vec{AB} = m\vec{AC}$ then A, B, C co-linear.

Proof : Since $\vec{AB} = m\vec{AC}$, $\vec{AB} \parallel \vec{AC}$. A and B lie on line l_1 and A and C lie on line l_2. Only one line can be drawn through a given point parallel to a given line, so l_1 and l_2 are the same line. therefore, A, B, C co-linear. $\square$

P42

[2] Checked.

[3.1] $3\vec{a} + 4\vec{a} - \vec{a} = (3+4-1)\vec{a} = 6\vec{a}$

[3.2] $3(\vec{u} + 2\vec{v}) - 3(\vec{u} - 4\vec{v})$

$= 3\vec{u} + 6\vec{v} - 3\vec{u} + 12\vec{v}$

$= 18\vec{v}$

[4] Let $\vec{a} = \vec{AB}$, $b = \vec{AC}$

$\vec{NL} = \frac{1}{2}\vec{b}$

$\vec{LM} = \frac{1}{2}(-\vec{a}) = -\frac{1}{2}\vec{a}$

$\vec{MN} = \frac{1}{2}\vec{a} - \frac{1}{2}\vec{b} = \frac{1}{2}(\vec{a} - \vec{b})$

$\vec{AL} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} = \frac{1}{2}(\vec{a} + \vec{b})$

$\vec{BM} = -\vec{a} + \frac{1}{2}\vec{b}$

$\vec{CN} = -b + \frac{1}{2}\vec{a}$

[5] check $\vec{e} = \frac{\vec{a}}{|\vec{a}|}$, $\vec{a} \neq \vec{0}$.

Prove that $\vec{e} = \frac{\vec{a}}{|\vec{a}|}$ is a unit vector with same direction as $\vec{a}$.

Proof

$\frac{1}{|\vec{a}|}$ is a scalar, call it m. so,

$\frac{\vec{a}}{|\vec{a}|} = m\vec{a}$. Thus, $\frac{\vec{a}}{|\vec{a}|} \parallel \vec{a}$. I.e. $\boxed{\vec{e} \parallel \vec{a}.}$

Now $|\vec{e}| = \left|\frac{\vec{a}}{|\vec{a}|}\right| = \frac{|\vec{a}|}{||\vec{a}||} = \frac{|\vec{a}|}{|\vec{a}|} = 1.$

Therefore $\vec{e} = \frac{\vec{a}}{|\vec{a}|}$ is a unit vector in the same direction as $\vec{a}$.

$\square$

P44

[1] $A(-2,-6)$, $B(3,1)$, $C(3,4)$

$\vec{AB} = 5\hat{e}_1 + 7\hat{e}_2$

$\vec{BC} = 3\hat{e}_2$

$\vec{CA} = -5\hat{e}_1 - 10\hat{e}_2$

P45

[2] $\vec{a} = -3\hat{e}_2$

$\vec{b} = -2\hat{e}_1$

$\vec{c} = 4\hat{e}_1 + 2\hat{e}_2$

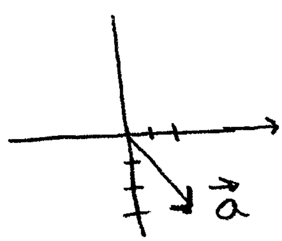
$\vec{d} = 3\hat{e}_1 - 3\hat{e}_2$

[3] $\vec{AB} = \langle 5, 3 \rangle$

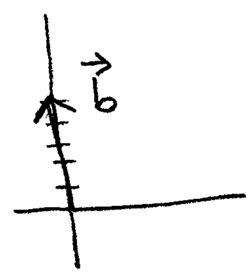
$\vec{BA} = \langle -5, -3 \rangle$

P46

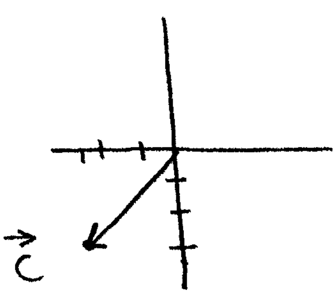
[4.1]



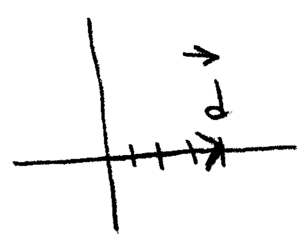
[4.2]



[4.3]



[4.4]



[5] a) Derive: $\langle a_1, a_2\rangle - \langle b_1, b_2\rangle = \langle a_1 - b_1, a_2 - b_2\rangle$

$$\vec{a} = a_1\hat{e}_1 + a_2\hat{e}_2$$

$$\vec{b} = b_1\hat{e}_1 + b_2\hat{e}_2$$

$$\vec{a} - \vec{b} = (a_1 - b_1)\hat{e}_1 + (a_2 - b_2)\hat{e}_2$$

$$= \langle a_1 - b_1, a_2 - b_2\rangle$$

b) derive $m\langle a_1, a_2\rangle = \langle ma_1, ma_2\rangle$

$$\vec{a} = a_1\hat{e}_1 + a_2\hat{e}_2$$

$$m\vec{a} = ma_1\hat{e}_1 + ma_2\hat{e}_2$$

$$= \langle ma_1, ma_2\rangle$$

[6.0] $\vec{a} = \langle 2, -3\rangle$ $\vec{b} = \langle -1, 2\rangle$ $\vec{c} = \langle 5, 0\rangle$

[6.1] $\vec{u} = \vec{a} + \vec{b} + \vec{c} = \langle 2, -3\rangle + \langle -1, 2\rangle + \langle 5, 0\rangle$

$$= \langle 2-1+5, -3+2+0\rangle$$

$$= \langle 6, -1\rangle$$

[6.2] $\vec{v} = -3\vec{a} + 2\vec{b} + \vec{c}$

$$= -3\langle 2, -3\rangle + 2\langle -1, 2\rangle + \langle 5, 0\rangle$$

$$= \langle -6, 9\rangle + \langle -2, 4\rangle + \langle 5, 0\rangle$$

$$= \langle -6-2+5, 9+4+0\rangle$$

$$= \langle -3, 13\rangle$$

$\rightarrow$

p47, ctd

[6.3] $\vec{w} = 3(\vec{a} + \vec{b}) - 2(\vec{b} - \vec{c})$

$= 3 \langle 1, -1 \rangle - 2 \langle -6, 2 \rangle$

$= \langle 3, -3 \rangle + \langle 12 - 4 \rangle$

$= \langle 3 + 12, -3 - 4 \rangle$

$= \langle 15, -7 \rangle$

[7] $\vec{a} = \langle 2, -3 \rangle$, $\vec{b} = \langle -1, 2 \rangle$, $c = \langle 5, 0 \rangle$

$|\vec{a}| = \sqrt{4 + 9} = \sqrt{13}$

$|\vec{b}| = \sqrt{1 + 4} = \sqrt{5}$

$|\vec{c}| = \sqrt{25 + 0} = 5$

p 49

[1] SHOW $\theta = 0°$, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos 0°$

$= |\vec{a}||\vec{b}|$

Show $\theta = 180°$, $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos 180$

$= |\vec{a}||\vec{b}| (-1)$

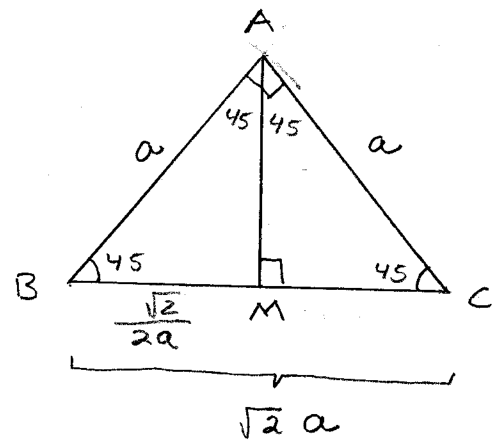
$= -|\vec{a}||\vec{b}|$

[2.1] $\overrightarrow{AB} \cdot \overrightarrow{AC} = 2\sqrt{3}\cos 30 = 2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = 3$

[2.2] $\overrightarrow{CA} \bullet \overrightarrow{CB} = 0$, $\because \angle C = 90^\circ$ and $\cos 90 = 0$

[2.3] $\overrightarrow{AB} \cdot \overrightarrow{BC} = -\overrightarrow{BA} \cdot \overrightarrow{BC} = -(2)(1)\cos 60 = -2\left(\frac{1}{2}\right) = -1$

[2.4] $\overrightarrow{AB} \bullet \overrightarrow{CA} = \overrightarrow{AB} \cdot (-\overrightarrow{AC}) = -2\sqrt{3}\cos 30 = -2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = -3$



[3.1] $\overrightarrow{AB} \bullet \overrightarrow{CB}$

$= -\overrightarrow{BA} \cdot (-\overrightarrow{BC})$

$= \overrightarrow{BA} \cdot \overrightarrow{BC}$

$= (a)(\sqrt{2}\,a)\cos 45$

$= a^2\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)$

$= a^2$

[3.2] $\overrightarrow{BA} \bullet \overrightarrow{CA} = -\overrightarrow{AB} \cdot (-\overrightarrow{AC}) = \overrightarrow{AB} \cdot \overrightarrow{AC} = 0$, $\because \angle A = 90^\circ$

[3.3] $\overrightarrow{AM} \cdot \overrightarrow{BA} = \overrightarrow{AM} \cdot (-\overrightarrow{AB}) = -\overrightarrow{AM} \cdot \overrightarrow{AB}$

$= -\left(\frac{\sqrt{2}}{2a}\right)(a)\cos 45^\circ$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}$

$= -\frac{1}{2}$

P 52

[4.1] $\vec{a} \cdot \vec{b} = (-2)(5) + (3)(4) = -10 + 12 = 2$

[4.2] $\vec{a} \cdot \vec{b} = (3)(-4) + (5)(1) = -12 + 5 = -7$

[5.0] $\vec{a} = \langle 3, -4 \rangle$, $\vec{b} = \langle 8, 6 \rangle$ Show $\vec{a} \perp \vec{b}$

$\vec{a} \cdot \vec{b} = (3)(8) + (-4)(6) = 0$, $\therefore \vec{a} \perp \vec{b}$

[6.1] $$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{(-3)(-1) + (0)(\sqrt{3})}{(3)(2)} = \frac{3}{6} = \frac{1}{2}$$

$$\cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \boxed{\theta = 60^\circ}$$

[6.2] $$\cos\theta = \frac{(2)(3) + (1)(-6)}{\sqrt{5} \cdot \sqrt{45}} = 0$$

$$\therefore \theta = 90^\circ$$

P 53

[7.1] Prove: $(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}$

<u>Proof</u>

$$\begin{aligned} LHS &= (\vec{a} + \vec{b}) \cdot \vec{c} \\ &= \langle a_1 + b_1, a_2 + b_2 \rangle \cdot \langle c_1, c_2 \rangle \\ &= (a_1 + b_1) c_1 + (a_2 + b_2) c_2 \\ &= a_1 c_1 + b_1 c_1 + a_2 c_2 + b_2 c_2 \\ &= a_1 c_1 + a_2 c_2 + b_1 c_1 + b_2 c_2 \\ &= \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} \end{aligned}$$

□

p 53, ctd

[7.2] Prove: $\vec{a} \cdot m\vec{b} = m(\vec{a} \cdot \vec{b})$

Proof

$$
\begin{aligned}
LHS &= \vec{a} \cdot m\vec{b} \\
&= \langle a_1, a_2 \rangle \cdot m \langle b_1, b_2 \rangle \\
&= \langle a_1, a_2 \rangle \cdot \langle mb_1, mb_2 \rangle \\
&= a_1 m b_1 + a_2 m b_2 \\
&= m(a_1 b_1 + a_2 b_2) \\
&= m(\vec{a} \cdot \vec{b}) \\
&= RHS
\end{aligned}
$$

□

[7.3] Prove: $(m\vec{a}) \cdot \vec{b} = m(\vec{a} \cdot \vec{b})$

Proof

$$
\begin{aligned}
LHS &= (m\vec{a}) \cdot \vec{b} \\
&= \langle m.a_1, ma_2 \rangle \cdot \langle b_1, b_2 \rangle \\
&= ma_1 b_1 + ma_2 b_2 \\
&= m(a_1 b_1 + a_2 b_2) \\
&= m(\vec{a} \cdot \vec{b}) \\
&= RHS
\end{aligned}
$$

□

[8.1] Prove $\vec{a} \cdot (\vec{b} - \vec{c}) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$

proof

{ NOTE: We just proved $(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}$, which says dot product is Right distributive, we'll prove dot product is left distributive for $(\vec{a} + \vec{b})$, then prove The immediate thm by merely substituting $-\vec{b}$ for $\vec{b}$. }

Prove $\vec{c} \cdot (\vec{a} + \vec{b}) = \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b}$

Proof

$$
\begin{aligned}
LHS &= \langle c_1, c_2 \rangle \cdot \langle a_1 + b_1, a_2 + b_2 \rangle \\
&= c_1(a_1 + b_1) + c_2(a_2 + b_2) \\
&= c_1 a_1 + c_1 b_1 + c_2 a_2 + c_2 b_2 \\
&= c_1 a_1 + c_2 a_2 + c_1 b_1 + c_2 b_2 \\
&= \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} \\
&= RHS
\end{aligned}
$$

$\square$

Now,

$$
\begin{aligned}
\vec{a} \cdot (\vec{b} - \vec{c}) &= \vec{a} \cdot (\vec{b} + (-\vec{c})) \\
&= \vec{a} \cdot \vec{b} + \vec{a} \cdot (-\vec{c}) \\
&= \vec{a} \cdot \vec{b} + \vec{a} \cdot (-1)\vec{c} \\
&= \vec{a} \cdot \vec{b} + (-1)\, \vec{a} \cdot \vec{c} \\
&= \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}
\end{aligned}
$$

$\square$

p 53, ctd

[8,2] Prove: $(\vec{a}-\vec{b})\cdot\vec{c} = \vec{a}\cdot\vec{c} - \vec{b}\cdot\vec{c}$

Proof

$$\begin{aligned} LHS &= (\vec{a}-\vec{b})\cdot\vec{c} \\ &= (\vec{a}+(-\vec{b}))\cdot\vec{c} \\ &= \vec{a}\cdot\vec{c} + (-\vec{b})\cdot\vec{c} \\ &= \vec{a}\cdot\vec{c} - \vec{b}\cdot\vec{c} \end{aligned}$$

$\square$

p 54

[9] Let: $|\vec{a}| = 2$, $|\vec{b}| = 3$, $\vec{a}\cdot\vec{b} = 4$

Find $|2\vec{a} - 3\vec{b}|$

SOLN

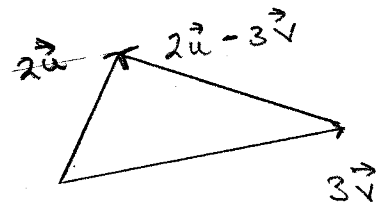
$$\begin{aligned} |2\vec{a}-3\vec{b}|^2 &= \langle 2\vec{a}-3\vec{b}\rangle \cdot \langle 2\vec{a}-3\vec{b}\rangle \\ &= 2\vec{a}\cdot\langle 2\vec{a}-3\vec{b}\rangle - 3\vec{b}\cdot\langle 2\vec{a}-3\vec{b}\rangle \\ &= 2\vec{a}\cdot 2\vec{a} - 2\vec{a}\cdot 3\vec{b} - 2a\cdot 3\vec{b} + 3\vec{b}\cdot 3\vec{b} \\ &= 4|\vec{a}|^2 - 12\,\vec{a}\cdot\vec{b} + 9|\vec{b}|^2 \\ &= 4(4) - 12(4) + 9(9) \\ &= 16 - 48 + 81 \\ &= 49 \end{aligned}$$

$\therefore\ |2\vec{a}-3\vec{b}| = \sqrt{49} = 7$

P 55

[1.1] $6\vec{u} - 5\vec{v} - 4\vec{u} + 2\vec{v}$

$= 2\vec{u} - 3\vec{v}$

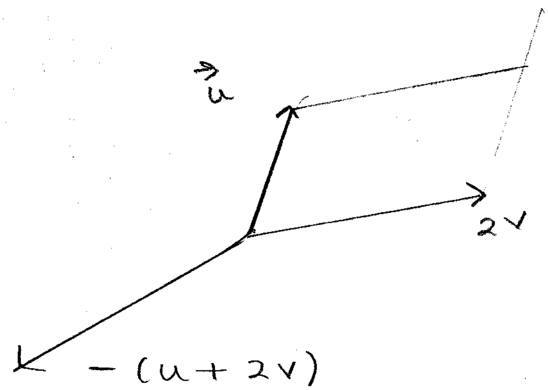


[1.2] $7(\vec{u} - 2\vec{v}) - 4(2\vec{u} - 3\vec{v})$

$= 7\vec{u} - 14\vec{v} - 8u + 12v$

$= -u - 2v$

$= -(u + 2v)$



[2] $\vec{c} = k\vec{a} + l\vec{b}$, $\vec{a} = \langle -2, 3\rangle$, $\vec{b} = \langle 1, -4\rangle$, $\vec{c} = \langle 8, -17\rangle$

SOLN

$k\vec{a} + l\vec{b} = \vec{c}$

$\langle ka_1, ka_2\rangle + \langle lb_1, lb_2\rangle = \langle 8, -17\rangle$

$\Rightarrow \begin{bmatrix} ka_1 + lb_1 = 8 \\ ka_2 + lb_2 = -17 \end{bmatrix}$

$\Leftrightarrow \begin{bmatrix} -2k + l = 8 \\ 3k - 4l = -17 \end{bmatrix} \Rightarrow \begin{bmatrix} -8k + 4l = 32 \\ 3k - 4l = -17 \end{bmatrix}$

$\Rightarrow -5k = 15 \Rightarrow \boxed{k = -3}$

$-2(-3) + l = 8$

$6 + l = 8$

$\boxed{l = 2}$

P 55, ctd

[3.1] Prove: $(4\vec{a} + 3\vec{b}) \bullet (4\vec{a} - 3\vec{b}) = 16|\vec{a}|^2 - 9|\vec{b}|^2$

Proof

$$LHS = (4\vec{a} + 3\vec{b}) \bullet (4\vec{a} - 3\vec{b})$$
$$= 16\,\vec{a}\bullet\vec{a} - 9\,\vec{b}\circ\vec{b}$$
$$= 16\,|\vec{a}|^2 - 9\,|\vec{b}|^2$$
$$= RHS$$

□

[3.2] Prove: $|\vec{a}+\vec{b}|^2 - |\vec{a}-\vec{b}|^2 = 4\vec{a}\bullet\vec{b}$

Proof

$$LHS = |\vec{a}+\vec{b}|^2 - |\vec{a}-\vec{b}|^2$$
$$= \langle\vec{a}+\vec{b}\rangle \bullet \langle\vec{a}+\vec{b}\rangle - \langle\vec{a}-\vec{b}\rangle \bullet \langle\vec{a}-\vec{b}\rangle$$
$$= \cancel{\vec{a}\cdot\vec{a}} + 2\vec{a}\cdot\vec{b} + \cancel{\vec{b}\cdot\vec{b}} - \cancel{\vec{a}\cdot\vec{a}} + 2\vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{b}$$
$$= 4\,\vec{a}\bullet\vec{b}$$
$$= RHS$$

□

[4.1] $|\vec{a}| = 3,\ |\vec{b}| = 4,\ \vec{a}\bullet\vec{b} = 6$

$$\vec{a}\bullet\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{6}{12} = \frac{1}{2}$$

$$\therefore\ \theta = 60^\circ$$

P55, ctd

[4.2] $\cos\theta = \frac{\sqrt{2}}{2} \Rightarrow \theta = 45°$

[5] $\vec{u} = 4x\langle 2,1\rangle + \langle -1,2\rangle$

$= \langle 8x, 4x\rangle + \langle -1,2\rangle$

$= \langle 8x-1, 4x+2\rangle$

$\vec{v} = x\langle 2,1\rangle - 3\langle -1,2\rangle$

$= \langle 2x, x\rangle + \langle 3,-6\rangle$

$= \langle 2x+3, x-6\rangle$

$\vec{u}\cdot\vec{v} = \langle 8x-1, 4x+2\rangle \bullet \langle 2x+3, x-6\rangle$

$= (8x-1)(2x+3) + (4x+2)(x-6)$

$= (16x^2+22x-3) + (4x^2-22x-12)$

$= 20x^2 - 15$

$= 5(4x^2-3)$

$\vec{u} \perp \vec{v}$ iff $\vec{u}\cdot\vec{v} = 0$

$5(4x^2-3) = 0$

$\Rightarrow \quad 4x^2 - 3 = 0$

$\Rightarrow \quad x = \pm\frac{\sqrt{3}}{2}$

□

checked answer.

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[6] Prove for $\vec{a} \neq 0, \vec{b} \neq 0, \quad |\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| \Rightarrow \vec{a} \perp \vec{b}$

Proof

suppose $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

$$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$

$$(\vec{a}+\vec{b}) \cdot (\vec{a}+\vec{b}) = (\vec{a}-\vec{b}) \cdot (\vec{a}-\vec{b})$$

$$\vec{a}\cdot\vec{a} + 2\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b} = \vec{a}\cdot\vec{a} - 2\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b}$$

$$4\vec{a}\cdot\vec{b} = 0$$

$$\vec{a}\cdot\vec{b} = 0$$

$$\therefore \vec{a} \perp \vec{b}$$

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[7] Show $|\vec{OA} \cdot \vec{e}| = OB$

Proof

call angle AOB angle θ.

$$|\vec{OA} \cdot \vec{e}| = \big| |\vec{OA}||\vec{e}| \cos\theta \big|$$

$$= |\vec{OA}| \, |\cos\theta|$$

$$= |\vec{OA}| \cos\theta \qquad , \cos\theta > 0 \because 0 < \theta < 90^\circ$$

$$= |\vec{OA}| \frac{|\vec{OB}|}{|\vec{OA}|}$$

$$= |\vec{OB}|$$

$\square$